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Generation–recombination noise due to localisation in spin-split Landau levels of a quantum Hall system

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Abstract. The existence of localised states in the tails of spin-split Landau levels was confirmed by noise spectroscopy performed on a two-dimensional electron gas in a GaAs/AlGaAs heterostructure. Noise was measured under integral quantum Hall conditions in the filling factor range $2 < \nu < 4$. In the spectra of the noise in the Hall voltage at least two Lorentzians could be distinguished. They were attributed to generation–recombination processes involving localised states in the tails of the spin-split Landau levels. In addition, in the range $2\frac{1}{2} < \nu < 3\frac{1}{2}$ a third Lorentzian was found, which provides evidence for the existence of localised states between the two spin levels. The magnetic field dependence of the Lorentzian noise intensities indicate that the localised states are distributed asymmetrically over a spin level.

1. Introduction

In recent years the electronic properties of two-dimensional electron systems have attracted considerable attention [1]. In particular, the discovery of the quantum Hall effect (QHE) [2] has stimulated experimental as well as theoretical research on magneto-transport in these systems. The QHE can be regarded as a macrosopic phenomenon in electrical conduction arising from Landau quantisation. In a two-dimensional electron gas (2DEG) subjected to a strong perpendicular magnetic field, the Hall resistance R_H exhibits a series of steps given by $R_H = h/\nu e^2$, where h is Planck's constant, e is the elementary charge and ν is either an integer (*integral* QHE) or a fraction (*fractional* QHE). Whereas the fractional effect is thought to arise from electron–electron interactions, the integral effect is generally believed to be characteristic of disordered systems in which electron–electron interactions are relatively unimportant. The energies ε_N of non-interacting electrons in a 2DEG in a magnetic field B are quantised according to

$$\varepsilon_N = (N + \tfrac{1}{2})\hbar\omega_c + sg\mu_B B \qquad (1)$$

where N is the Landau level index, $\omega_c = eB/m^*$ is the cyclotron angular frequency, s is the spin quantum number, g the Landé factor, μ_B is the Bohr magneton and m^* is the effective mass of the electrons in the two-dimensional system. Electron scattering at the interface potential, however, broadens the Landau levels [3]. As long as the Fermi level is pinned to localised states in the gap between adjacent Landau levels [4], quantisation of the Hall resistance can occur, provided the width of the Landau levels and the thermal energy kT are small compared to the Landau level separation $\hbar\omega_c$. Very direct experimental evidence for the existence of such localised states was provided by recent electrical noise measurements on GaAs/AlGaAs heterostructures under integral QHE conditions [5–7]. Fluctuations in the number of electrons occupying extended states gave rise to Lorentzian voltage noise spectra. The fluctuations were attributed to electronic transitions into and out of localised states in the tails of the Landau levels (generation–recombination or GR noise). The experimental data were compared with calculations of the GR noise due to these intra-Landau level transitions under the assumption that all states were spin degenerate. In these experiments the magnetic field strength was not sufficiently high to completely resolve the Zeeman splitting of the two spin levels pertaining to the Landau level involved in the fluctuation processes, although at the upper limit of the magnetic field range the onset of the lifting of the spin degeneracy was visible in the Shubnikov–de Haas (SDH) oscillations. Therefore, in the spin-split regime we expected transitions between the two spin levels to affect the spectral noise in the Hall resistance and in the magnetoresistance. This assumption was confirmed by the discrepancy between calculations and experimental data of the noise in the regime where the spin splitting was apparent. Thus the numerical analysis of the noise data was found to be valid only when the Fermi level was located in a *spin-unresolved* Landau level.

In this paper we present an experimental study of the electrical noise in GaAs/AlGaAs heterostructures under conditions suitable for the observation of the integral QHE. The noise measurements were performed in magnetic fields and at temperatures where the spin splitting of the Landau level in which the Fermi level lies is resolved. On the basis of earlier noise experiments, fluctuations due to transitions between different spin levels are expected. The primary goal is to determine the character and origin of electrical fluctuations under these experimental circumstances. Since noise spectroscopy is a powerful tool for investigating localisation properties, these measurements may contribute to our understanding of electrical conduction under quantum Hall conditions. In particular our objective is to test the viability of the localisation concept, which is usually invoked to explain the integral QHE. In §2 the experimental results are presented following a brief description of the set-up for noise measurements. The results of the noise measurements will be discussed in §3.

2. Experimental

The samples studied were GaAs/$Al_xGa_{1-x}As$ heterostructures ($x = 0.34$). The active GaAs layer was separated from a silicon-doped AlGaAs layer ($N_{Si} = 1.3 \times 10^{23}$ m^{-3}) by a 100 Å undoped AlGaAs spacer layer. We used heterostructures with a standard Hall-bar geometry of dimensions 150×1100 μm^2. The longitudinal or SDH voltage was measured across two contacts that were 400 μm apart. At $T = 0.6$ K the electron concentration n_e in the 2DEG was 5.0×10^{15} m^{-2} and the electron mobility μ was 10.2 m^2 V^{-1} s^{-1}. The devices were mounted in a ^{3}He immersion cryostat equipped with a superconducting magnet. Magnetic fields up to 10 T could be applied at right angles to the 2DEG. A constant DC current was passed through the 2DEG. The noise in the Hall voltage was amplified by an ultra-low-noise amplifier. A frequency analysis of the amplified signal was made by means of fast Fourier transformation, yielding a power spectrum ranging from 0.5 Hz to 100 kHz. The noise of the unbiased sample as well as the noise of the amplifier were subtracted from the noise measured on the biased sample. Thus the spectral intensity of the excess noise of the 2DEG was obtained. A typical recording of the DC Hall voltage V_H and of the DC SDH voltage V_L in the magnetic field range 4 to 10 T is shown in figure 1. The number of filled spin split levels (or filling factor ν, defined by $\nu = n_e h/eB$) is varied by varying the magnetic induction B while the electron concentration n_e is kept constant. In figure 1 the range of filling factors covered by the noise measurements is indicated. It is clear from this figure that the spin splitting of the second Landau level ($N = 1$) is resolved. The resolution however is not complete since the minimum in V_L at $\nu = 3$ does not drop to zero; this points to the occurrence of scattering between the spin-down level 1↓ and the spin-up level 1↑. The $\nu = 3$ minimum was observed to decrease as the current I through the 2DEG was decreased. Electron heating induced by the electric field is likely to be responsible, since in a SDH minimum the longitudinal voltage is known to be thermally activated [8].

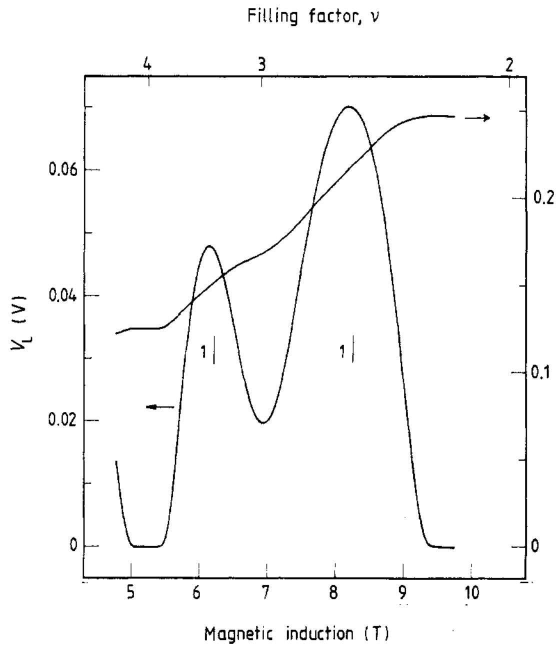

Figure 1. The longitudinal voltage V_L and the Hall voltage V_H versus the magnetic induction in the range $2 < \nu < 4$, measured at 0.6 K. The current through the 2DEG was 20 μA. 1↓ and 1↑ denote the coincidence of the Fermi level with the spin-down level and the spin-up level respectively of the Landau level with index $N = 1$.

In figure 2 a typical spectrum of the excess noise $S_{V_H}(f)$ in the Hall voltage is plotted. The spectrum was taken at $B = 8.8$ T where V_L is intermediate between the $\nu = 2$ minimum and the $\nu = 2\frac{1}{2}$ maximum. A strong frequency dependence was found in the noise. A sum of three Lorentzians could be fitted to the experimental data of this particular spectrum:

$$S_{V_H}(f) = \sum_{i=1}^{3} \frac{S_i(0)}{1 + \omega^2 \tau_i^2} \tag{2}$$

where the angular frequency $\omega = 2\pi f$. The index i refers to one of the three Lorentzians and likewise identifies the Lorentzians which are indicated separately in the spectrum of figure 2. The computer fit yielded the Lorentzian parameters $S_i(0)$, which is the zero-frequency intensity of Lorentzian i, and τ_i, which is a relaxation time that is the reciprocal of the angular corner frequency of a Lorentzian. The τ_i are associated with the relaxation of fluctuations to the stationary state. From the measured spectra τ_1, τ_2 and τ_3 were found to be of the order of 10^{-1} s, 10^{-3} s and 10^{-5} s respectively.

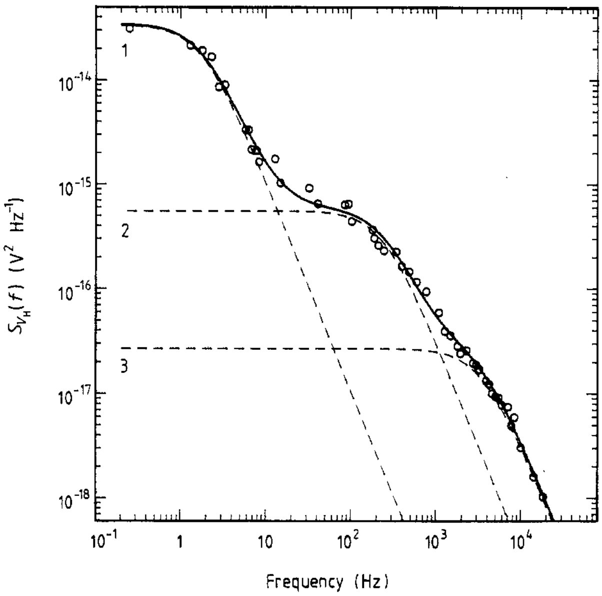

Figure 2. Spectral intensity $S_{V_H}(f)$ of the excess noise in the Hall voltage, measured at 0.6 K and at $B = 8.8$ T. The current I through the 2DEG was $I = 20$ μA. The full curve is a fit of equation (2) to the experimental data (open circles). The three Lorentzians are indicated separately by broken curves. The labels 1, 2 and 3 refer to the three Lorentzians.

It was found that for $I < 25$ μA all three relaxation times were independent of I, whereas the corresponding Lorentzian noise intensities varied quadratically with I in the entire frequency range considered.

In earlier experiments on electrical noise in quantum Hall systems with spin-degenerate Landau levels [6, 7] generally two Lorentzians were found in the spectra of the noise in the Hall voltage and in the longitudinal voltage. Each Lorentzian was found to depend on the magnetic field strength. For this reason we now investigate the magnetic field dependence of the spectral noise of the present system in which the spin degeneracy is lifted. All spectra in the magnetic field range under consideration show a similar frequency dependence, although at magnetic fields below 6.0 T and above 9.5 T only two Lorentzians can be distinguished clearly, whereas at intermediate fields a third Lorentzian ($\tau \sim 10^{-5}$ s) is apparent.

The relaxation times τ_i as obtained from the fitting to the noise spectra are shown in figure 3. The error bars in the data points indicate the error that resulted from the fit. In fact, this error is a lower limit of the total experimental error, since the scattering of the data points is much larger than might be expected from the magnitude of the error bars. The variation in τ_i resulting from repeated measurements of the spectral noise was indeed much larger than the fitting error. The precise origin of the large experimental error could not be determined, although we surmise that it is due to the extreme sensitivity of the system to mechanical disturbances. The experimental error is estimated to be 25%.

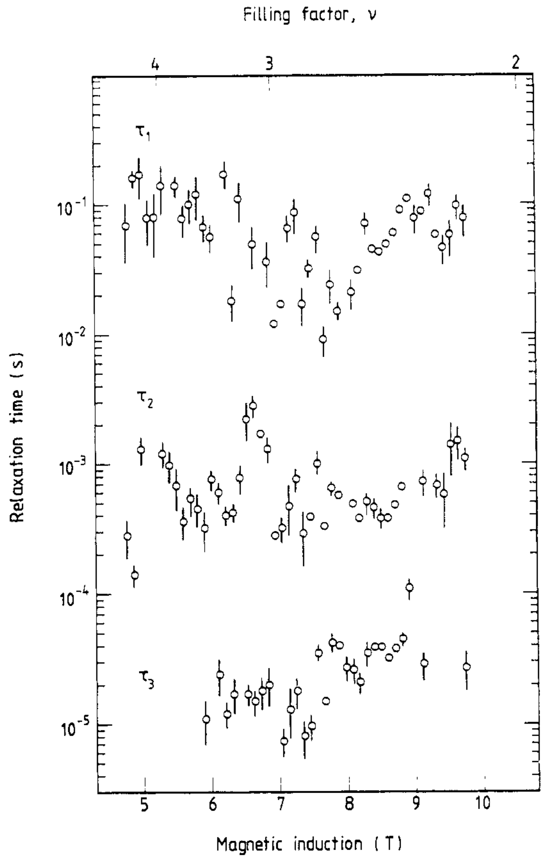

Figure 3. Relaxation times τ_i of the Lorentzians versus the magnetic induction. The subscript i corresponds to the labels 1,2, and 3 in figure 2.

A pronounced dependence of the relaxation times on the magnetic induction (or the filling factor) is apparent. Over the total magnetic field range the variation of the relaxation times is about one order of magnitude. Furthermore the τ_i show maxima and minima. However, experimental uncertainty hampered a detailed analysis of the magnetic field dependence of the τ_i. The intensity of the Lorentzians $S_i(0)$ showed a much more distinctive behaviour as a function of the filling factor, which can be seen from figure 4. The $S_i(0)$ are obtained from fits to the spectra. If experimental errors are taken into account, Lorentzian 1 has two significant maxima, namely at $B = 6.3$ T ($\nu \simeq 3\frac{1}{2}$) and at $B = 8.6$ T ($\nu \simeq 2\frac{1}{2}$). The second Lorentzian shows broad maxima at the same values of the magnetic induction as the first Lorentzian. However, a weak dip can be observed in the centre of these maxima. Note that $S_2(0)$ varies over two orders of magnitude. Furthermore Lorentzian 2 has a (weaker) maximum at $B = 5$ T for which $4 < \nu < 5$.

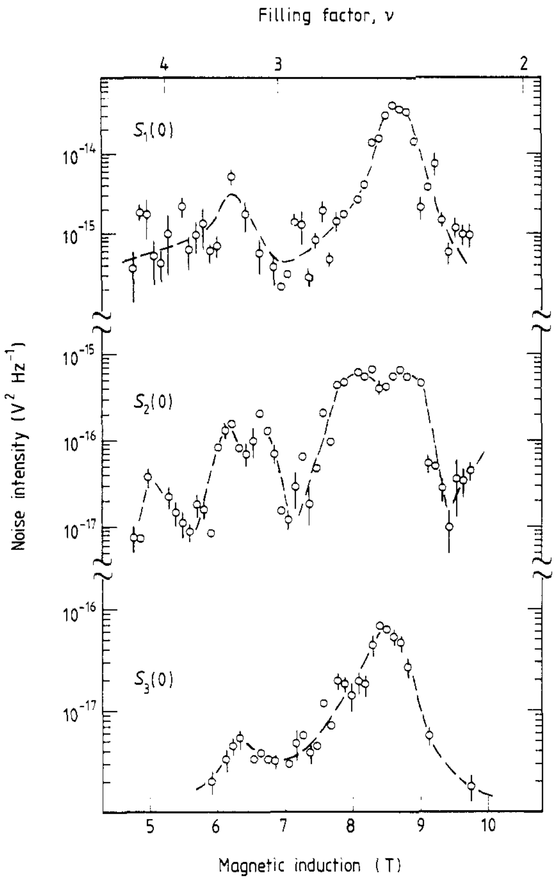

Figure 4. Intensities $S_i(0)$ of the Lorentzians (i=1,2,3) at zero frequency versus the magnetic induction. The broken curves are drawn to guide the eye.

So far the results are qualitatively similar to those of the earlier noise measurements. In the noise spectra taken in the range $B = 6.0$ to 9.5 T, however, a third Lorentzian can be distinguished. A strong maximum in the zero-frequency intensity of this Lorentzian, $S_3(0)$, is visible at $B = 8.4$ T or $\nu \simeq 2\frac{1}{2}$. A minor maximum appears at $B = 6.3$ T. For $B < 6.0$ T the third Lorentzian cannot be distinguished from the background noise. Furthermore it is apparent from the strong decrease in $S_3(0)$ for $B > 8.4$ T that Lorentzian 3 disappears for $B > 10$ T.

Summarising, we observed two or three Lorentzians having a maximum intensity at filling factors of approximately $i+\frac{1}{2}$, where i is an integer. Conversely, minima in the intensities of the three Lorentzians almost coincide with minima in the SDH voltage, which occur at integral filling factors.

3. Discussion

The Lorentzians can be ascribed to fluctuations in the number of free electrons, i.e. in the number of occupied extended states, since they were found to be quadratically dependent on the current. Free electrons can be temporarily trapped in localised states and trapped electrons can be temporarily added to the number of free electrons. In this way fluctuations in the resistance arise. The power spectrum of these fluctuations has an I^2 dependence, provided I does not fluctuate and Ohm's law applies. Previously, GR noise observed in a spin-degenerate system could be accounted for in a quantitative way by assuming that the states in the two tails of the Landau levels are localised [7]. Thus there are effectively three electronic reservoirs associated with a single Landau level; two of these contain localised states that are separated by mobility edges from extended states in the centre of the Landau level. Since electrical transport is determined by the electrons in the extended states, only fluctuations in the number of electrons in extended states are considered. In general, fluctuations of the number of occupied states in one reservoir due to transitions to and from n other reservoirs give rise to n Lorentzians if (i) intra-reservoir transitions are assumed to be very rapid compared to the noise relaxation times [9] and (ii) the total number of electrons in the $n+1$ reservoirs is constant. This means that in the case of a spin-degenerate Landau level two Lorentzians are expected; these were indeed found experimentally.

However, if the spin degeneracy is partially lifted, the density of states becomes more complicated. Consider a system consisting of two spin levels that belong to the same Landau level. Furthermore, assume that the states in the tails of each of the two spin levels are localised. This results in a system with at most six reservoirs, two of which contain extended states. If we assume that there is an energy overlap of the two localised states' reservoirs between the spin levels, then the system contains five reservoirs, namely the low-energy tail of $1\uparrow$, the region of extended states of $1\uparrow$, the region of extended states of $1\downarrow$, the high-energy tail of $1\downarrow$ and, finally, the region of localised states in between the two reservoirs of extended states. Such a system would in principle give rise to four Lorentzians in the noise spectra of the occupancy of each of the five reservoirs. However, in the experiment one measures fluctuations in the *total* number of extended electrons and thereby fluctuations in the sum of the occupancies of the two reservoirs of extended states. If N_1 and N_2 denote the numbers of electrons in the two reservoirs of extended states, the spectral noise intensity S_N of the total number $N = N_1 + N_2$ of extended electrons in the Landau level under consideration is given by

$$S_N = S_{N_1} + S_{N_2} + 2\mathrm{Re}(S_{N_1N_2}) \tag{3}$$

where $S_{N_1N_2}$ is the cross spectral noise intensity of N_1 and N_2. The last term in (3) will give a negative contribution to S_N. This follows directly from the definition of $S_{N_1N_2}$:

$$S_{N_1N_2} \propto \Delta N_1(\omega)\Delta N_2(\omega)^* \tag{4}$$

where $\Delta N_i(\omega)$ is the Fourier transform of the fluctuation $\Delta N_i(t)$ in the number N_i and the asterisk denotes complex conjugation. Fluctuations in N_1 and in N_2 are partly due to direct transitions between the corresponding reservoirs of extended states. This means that, with restriction to these particular fluctuations, $\Delta N_1 = -\Delta N_2$. Therefore a negative correlation exists between ΔN_1 and ΔN_2, which, according to (3) and (4), contributes negatively to S_N. This may explain why in the experiment only three Lorentzians were observed. Another, quite different reason for the absence of the fourth Lorentzian in our noise spectra may be the limited frequency range of the measurements or the fact that one of the Lorentzians is too small to be detected.

At a magnetic induction of $B = 5$ T the two spin levels $1\downarrow$ and $1\uparrow$ are completely filled, whereas the $2\downarrow$ level is completely empty, so GR noise is (almost) absent.

Increasing B partially depopulates the $1\downarrow$ level, thus permitting transitions to occur only in this level, because the $1\uparrow$ level is still completely filled. Since one spin level contains three electronic reservoirs, these transitions give rise to no more than two Lorentzians, which were indeed observed at low magnetic fields. A further increase in B depopulates the $1\uparrow$ level as well. Then, transitions between more than three partially filled reservoirs give rise to more than two and at most four Lorentzians. When subsequently B is increased, the $1\downarrow$ level becomes completely depopulated, restricting fluctuations to the $1\uparrow$ level. Hence again only two Lorentzians are observed. When B is increased still further, both spin levels become totally unoccupied, so no GR noise is seen in this regime.

As pointed out before, the third Lorentzian is only present in the magnetic field range between the two maxima of V_L, or, in other words, when the Fermi level lies in or between the two spin levels. This observation provides strong experimental evidence for the presence of localised states between the two spin levels. Moreover if $\nu > 4$ and the Fermi level lies in the Landau level with index $N = 2$, the intensities $S_1(0)$and $S_2(0)$ are substantial and show a maximum at B =5 T, whereas $S_3(0)$ does not rise above the background noise. These experimental facts indicate that the third Lorentzian is indeed associated with trapping involving localised states between resolved spin levels. This is corroborated by the fact that the spin splitting of $N = 2$ Landau level is unresolved, as was observed in the SDH oscillations. Thus only two Lorentzians are expected if the Fermi level lies in this Landau level.

The occurrence of maxima and minima in the intensity of the Lorentzians can be understood qualitatively in the following way. In [5] we pointed out that the *single* Lorentzian observed in the Hall voltage noise was at a maximum if the Fermi level is near a mobility edge, since then rates of transitions between localised states and extended states are at a maximum. For this reason maxima in $S_i(0)$ can be expected whenever the SDH voltage V_L has a value intermediate between a maximum and a minimum, or, whenever the filling factor ν of the spin-split system lies between i and $i + \frac{1}{2}$, where i is an integer. Minima are expected to coincide with minima in V_L (at integral ν), since the transition rates will be at a minimum if an integral number of spin levels are filled. However, in the noise spectra reported here we observed *three* Lorentzians. We point out that the intuitive interpretation, whereby each Lorentzian is identified with trapping in one particular reservoir of localised states, is incorrect, since the temperature is high enough to allow interactions between all reservoirs in the two spin levels. This hampers a simple analysis, such as was made in [5] of the Lorentzian noise intensities in terms of the position of the Fermi level with respect to the mobility edges. Nevertheless, we shall analyse our present results qualitatively.

Minima in all $S_i(0)$ are indeed observed at integral filling factors. The predicted behavior of the maxima in the GR noise is seen in Lorentzian 2. However, the minima in $S_2(0)$ expected at magnetic fields where V_L is at a maximum (at 6.4 T and 8.3 T) are just shallow dips, indicating that the distance between the mobility edges within one spin level is of the order of kT. Then, if the Fermi level ε_F is swept through the reservoir of extended states, the energy difference $\Delta\varepsilon = \varepsilon_F - \varepsilon_m$ is not much larger than kT, so due to Fermi–Dirac statistics, fluctuations in the number of occupied extended states can still occur. If the full width of the (Gaussian-shaped) spin levels is taken to be 1 meV [10], we can estimate that the distance between the mobility edges at T =0.6 K is of the order of kT, which is approximately 5% of the level width.

If it is assumed that the spin levels are symmetric and that the two mobility edges are located symmetrically with respect to the centre of the spin level, one would expect the dips in the maxima in $S_2(0)$ versus B to be observed at exactly those values of the magnetic induction where V_L is at a maximum. Surprisingly the dips are shifted over $\Delta B \simeq 0.3$ T to higher B. A tentative explanation may be that the two mobility edges of one spin level are asymmetrically positioned with respect to the centre of the spin level. If so, a dip in the intensity of the second Lorentzian occurs if the Fermi level lies halfway between the two mobility edges, where, due to the asymmetry, the density of states of the spin level is *not* at a maximum. The maxima in V_L, however, occur if the density of states at the Fermi level is at a maximum [11] and are therefore shifted with respect to the dips in $S_2(0)$. Since the dips occur at *higher* magnetic fields than the maxima in V_L, we conclude that more states are localised in the high-energy tail of a spin level than in the low-energy tail. An asymmetric shape of the spin levels [12, 13] might also result in a shift of the dips. On the basis of the present measurements, however, we cannot decide which of the two asymmetries mentioned is actually responsible.

Concerning Lorentzian 1 we point out that below B =7 T the data points of $S_1(0)$ are too scattered for us to draw firm conclusions. The pronounced maximum at B =8.6 T lies intermediate between a maximum and a minimum in V_L, as might be expected. The less striking structures in $S_1(0)$ however cannot be interpreted using the qualitative picture sketched above.

In order to make a thorough analysis of our experimental data, the model used for the calculation of GR noise in a spin-degenerate Landau level system [7] should be extended. Transitions between spin levels pertaining to the same Landau level must be incorporated. The analysis may become quite elaborate, because the GR noise due to transitions between at most six reservoirs must be calculated as a function of the Fermi energy ε_F, which in turn is determined by B and the shape of the spin-split Landau levels. On the basis of experimental evidence [14] the shape of a spin level can be taken to be Gaussian superimposed on a constant background density of states. Since the Zeeman splitting of the spin levels was found to be dependent on the filling factor ν [15, 16] and thereby on the Fermi level, the position of the Fermi level must be calculated self-consistently. If the model is adjusted to the noise data a more accurate estimate of the energy ranges of localised states in spin-split Landau levels can be obtained.

In summary, we have presented measurements of the spectral noise in the Hall voltage of a quantum Hall system in which the spin splitting of the $N = 1$ Landau level is resolved. In all spectra of the excess noise two Lorentzians were observed. When the Fermi level was located in or between the spin-up level and spin-down level, a third Lorentzian was found. The latter was not present when the spin splitting was not resolved, which demonstrates that the third Lorentzian originated from trapping of free electrons in localised states between two spin levels of the same Landau level. The anomaly in the positions of the maxima in $S_2(0)$ versus B indicates that the localised states in a spin level are possibly distributed in an asymmetric way with respect to the centre of the level.

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